TONBRIDGE SCHOOL

Scholarship Examination 2007

MATHEMATICS II

Wednesday 2nd May 2007
2.00 p.m.

Time allowed: 1 hour 30 minutes

Answer as many questions as you can.
All the questions carry equal marks.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

1. A large rectangle is made up of seven *identical* smaller rectangles as shown in the diagram below. If the area of the large rectangle is 525 cm², find the length and width of the small rectangles.
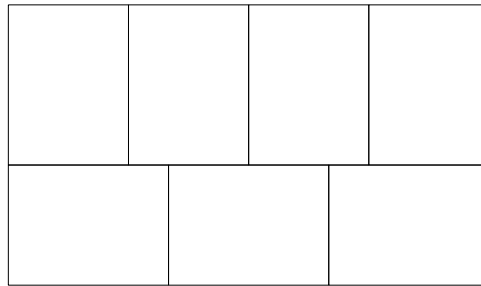
2. (a)   By first adding all the equations together, or otherwise, find the values of *A*, *B*, *C* which satisfy the equations below: the answers are not necessarily whole numbers!

    $$A + B + 2C = 21$$
    $$A + 2B + C = 22$$
    $$2A + B + C = 23$$

    (b)   Find the values of *A*, *B*, *C*, *D* which satisfy the equations:

    $$A + B + C + 2D = 21$$
 $$A + B + 2C + D = 22$$
    $$A + 2B + C + D = 23$$
    $$2A + B + C + D = 24$$

3. In this question, you will need to use the following formulae:
    - The volume of a hemisphere of radius *r* is $\frac{2}{3}\pi r^3$.
    - The volume of a cylinder of radius *r* and height *h* is $\pi r^2 h$.
    - The volume of a cone of radius *r* and perpendicular height *h* is $\frac{1}{3}\pi r^2 h$.

    The diagram depicts a hollow glass container with very thin walls made up of a hemisphere of radius 5 cm, a cylinder of radius 5 cm and height 4 cm, and a cone of radius 5 cm, perpendicular height 6 cm. Exactly half of the total volume of the container is filled with water.
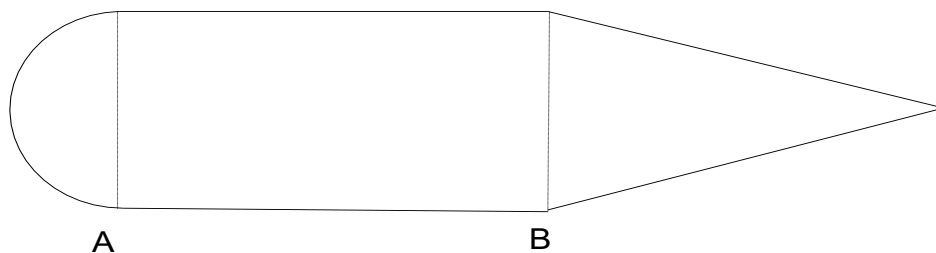

    (a)   If the container is held with B vertically above A, find the distance from A to the surface of the water.

    (b)   If the container is held with A vertically above B, find the distance from B to the surface of the water.

4. The figure enclosed by the solid lines below consists of a small semicircle with diameter AB of length 6 cm, together with equal arcs AC and BD of a larger semicircle with centre O and diameter COD of length 12 cm.
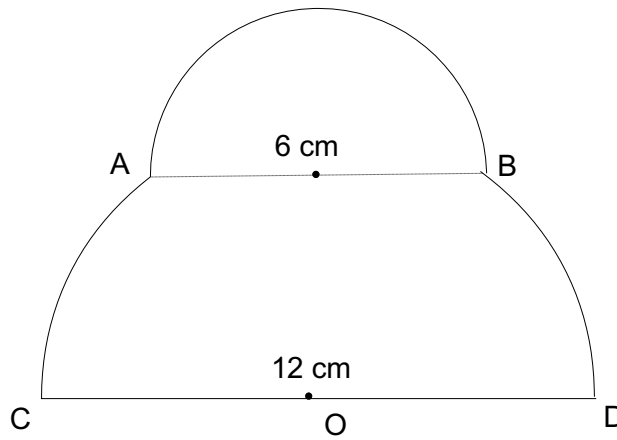
(a) Explain carefully why AOB is an equilateral triangle.

(b) Find the total area of the figure.

(c) Find the total perimeter of the figure.

5. In the diagram below, ABCD is a quadrilateral with angle ABC = angle CDA = 90° and angle DAB = 45°. Point E is on AD with EB = BD as shown; angle BED = x, angle ABE = y and angle DBC = z.
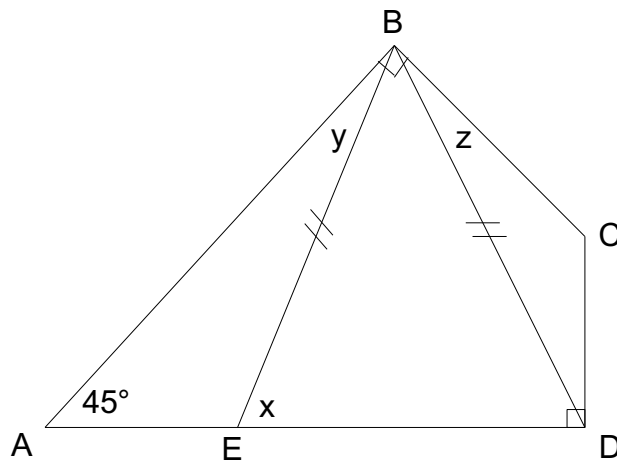
(a) If $x = 70°$, find y and z.

(b) Using algebra, show that, for any angle x, angles y and z are equal. Make your reasoning as clear as you can.

6.   For a number x, the notation $|x|$ is defined as follows:

- For a positive number, $|x|$ is x;
- For a negative number, $|x|$ is $-x$.

For example, $|5.2| = 5.2$ and $|-3.1| = 3.1$. Also, if you knew that $|x| = 4.5$, you could deduce that x is either 4.5 or -4.5.

In this question, you are asked to draw four graphs. Do them on separate axes, each with both x- and y-axes labelled from –3 to 3. For each graph, think very carefully about which positive and negative coordinates you should plot in view of the definition of $|x|$ given above.

(a)   Draw the graph of $y = |x|$.

(b)   Draw the graph of $|y| = |x|$.

(c)   Draw the graph of $y = \left| (|x| - 1) \right|$.

(d)   Draw the graph of $|y| = 3 - |x|$.

7.   In the table below, there is a connection between the columns A, B, C, between columns ABC and DEF and between columns DEF and GHI.

	A	B	C	D	E	F	G	H	I
Row 1	1	2	3	1	4	9	1	16	81
Row 2	2	3	5	4	9	25	16	81	625
Row 3	3	4	7	9	16	49	81	256	2401
Row 4	4								
Row 5	5								
Row n									

(a)   Study the table carefully and then write down the values of B, C, D, E, F, G, H, I corresponding to the blank entries in Row 4 and Row 5.

(b)   Write down the values of A, B, C, D, E, F, G, H, I for Row n, expressing your answers in terms of n.

(c)   Make a list of the values of $(D+E+F)^2$ and $G+H+I$ for Rows 1, 2, 3, 4, 5. What do you notice about your results?

(d)   Write down the formula involving n that results from applying your observation in (c) to Row n.